Introduction to Artificial Intelligence

Problem definition, State Space, Uninformed Search

Ing. Tomas Borovicka

Department of Theoretical Computer Science (KTI), Faculty of Information Technology (FIT)
Czech Technical University in Prague (CVUT)

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Problem Definition

- **Goal formulation**
  - Set of states that satisfies the agent (or maximizing some performance measure of the agent).
- **Problem formulation**
  - Process of deciding what actions and states to consider.

The Environment:

- Observable vs. partially observable vs. unobservable?
- Episodic vs. sequential?
- Discrete vs. continuous?
- Deterministic vs. stochastic?
- Static vs. dynamic?
- Known vs. unknown?

Problem Solving Agent

We assume that the environment is

- observable,
- discrete,
- deterministic,
- static.

- Under these assumptions, the solution to any problem can be expressed as a fixed **sequence of actions**.
- The process of looking for a sequence of actions that reaches the goal is called **search**.
- A search algorithm takes a problem as input and returns a solution in the form of an action sequence.
- Once a solution is found, the actions it recommends can be executed.

Definition of the State Space

- A problem can be defined by state space.
- The state-space is the configuration of the possible states and how they connect to each other e.g. the legal moves between states.

Definition (State Space)

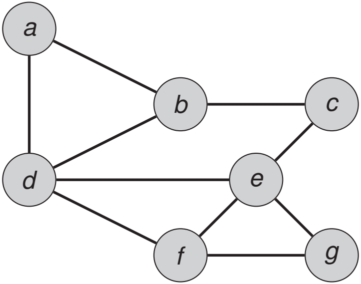
A state-space is a tuple $\langle S, A, \mathcal{I}, G \rangle$, such as (S, A) is a directed graph with a set of nodes S representing states and a set of edges A representing actions, $\mathcal{I}$ is then a set of initial states and G is a set of goal states.

- **State:** Some configuration of the problem.
- **Initial State:** The state in which the agent is at the beginning.
- **Actions:** Description of possible actions (moves) from the state s. Available actions might be a function of the state s $Actions(s) = \{a_1, \ldots\}$.
- **Goal State:** The state with certain properties (passes the a goal function).

A path in the state space is a sequence of states connected by a sequence of actions.

Graph Theory

- The graph theory is a branch of mathematics highly utilized in computer science. However the graph theory has a myriad of applications in many different domains.
- It is a study of graphs, where **graph** is a structure formed by **vertices** and **edges** connecting the vertices.
- Maps, websites, communication networks, electrical circuits, program structures, etc are practical examples of graphs.



Undirected Graphs

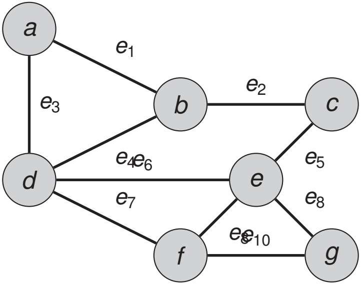
Definition (Undirected Graph)

Graph is a pair of sets (V, E) where V is the set of vertices and E is the set of edges, $E \subseteq \{\{u, v\} | u, v \in V\}$.

Example:

$V = \{a, b, c, d, e, f, g\}$

$E = \{\{a, b\}, \{b, c\}, \{a, d\}, \{b, d\}, \{c, e\}, \{d, e\}, \{d, f\}, \{e, f\}, \{e, g\}, \{f, g\}\}$

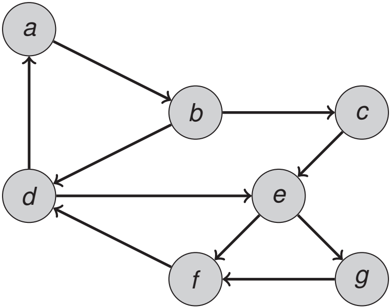


Directed Graphs

Definition (Directed Graph)

Directed graph is a pair of sets (V, E) where V is the set of vertices and E is the set of edges, such as $E \subseteq \{(u, v) | u, v \in V\}$, where (u, v) is an ordered pair, i.e. $(u, v) \neq (v, u)$.

- Each edge has a direction, i.e. (u, v) is an edge directed from node u to node v.



Graph Terminology 1

- A **walk** in the graph $G = (V, E)$ is a finite sequence $v_0, e_1, v_1, \ldots, v_k$ of vertices v_i and edges e_i such that for $1 <= i <= k$, the edge e_i has endpoints $v_{(i-1)}$ and v_i.
  - if $v_0 \neq v_k$ it is open walk, otherwise it is closed.
- A **trail** is a walk $v_0, e_1, v_1, \ldots, v_k$ with no repeated edge.
- A trail is a **path** if any vertex is visited at most once except possibly the initial and terminal vertices.
  - A **Hamiltonian path** is a path that includes all vertices of G.
- A **cycle** (or circuit) of a graph G is a subset of the edge set of E that forms a path such that the first node of the path corresponds to the last i.e. $v_1 = v_k$.
  - A cycle that uses each vertex of a graph exactly once is called a **Hamiltonian cycle**.

Similarly for directed graphs...

Graph Terminology 2

- Having directed graph $G = (V, E)$, if there is an edge from vertex u to vertex v, then u is a (direct) **predecessor** of v and v is a (direct) **successor** of u. We denote $\Gamma(u)$ as a **set of successors** of vertex u.
  $\Gamma(u) = \{v \in V | (u, v) \in E\}$.
- Connected acyclic graph is called a **tree** (can be directed or undirected).
- A tree with one special singled out vertex labeled "root" is called a **rooted tree**.
- A vertex in a directed tree such that $\Gamma(u) = \{ \}$ is called a **leaf**.

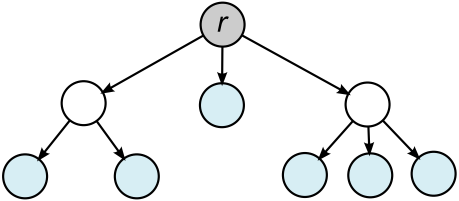


Figure: The Rooted Tree (with blue colored leafs)

Route Finding

The goal is to find a path from city *A* to city *B*.

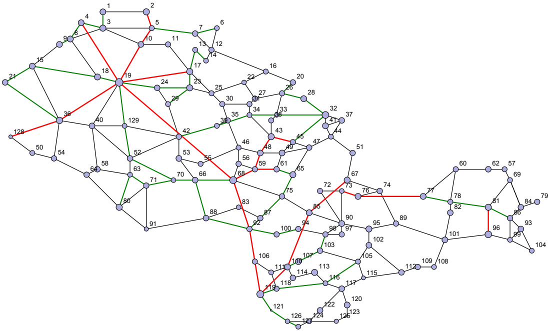
States: The city we are currently in.

Initial State: The city in which we start.

Actions: Moving from current city to adjacent cities.

Goal: City we want to reach.

- Intuitive State Space is then the road map:



Sliding-block Puzzles

Game consists of $N \times N$ board with $N - 1$ numbered tiles and a blank space. A tile adjacent to the blank space can slide into it. The goal is to reach specified order.

States: Description of the location of each tile and the blank square.

Initial State: Initial configuration of the puzzle.

Actions: Moving the blank left, right, up, or down.

Goal: Predefined configuration of the puzzle (ordering).

Example for $N = 4$:

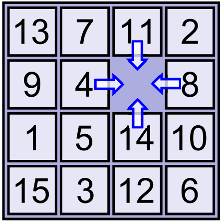


Figure: The Initial State

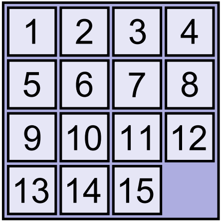
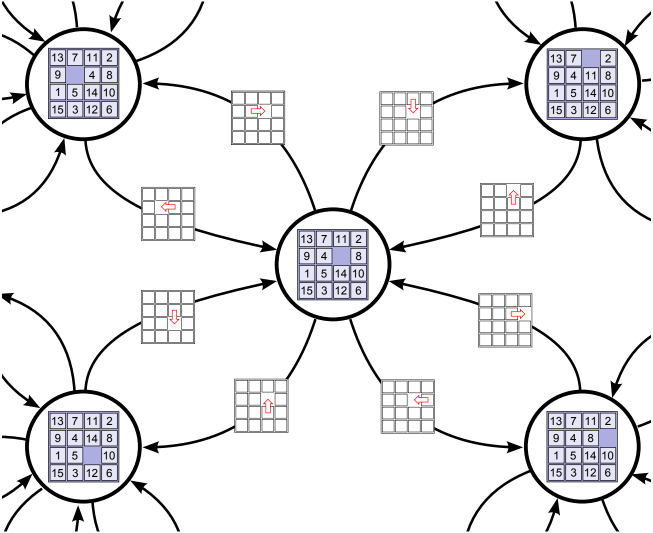
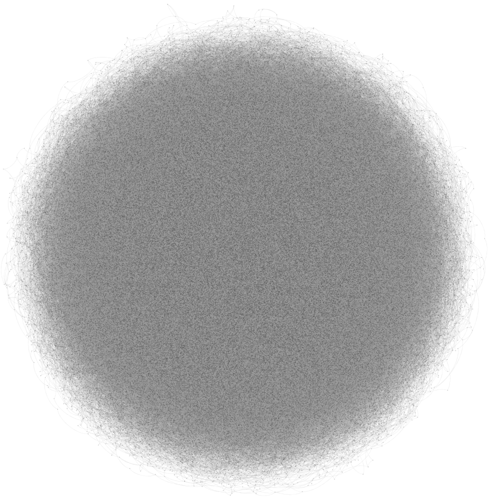


Figure: The Goal State

Sliding-block Puzzles: State Space $N = 4$



Sliding-block Puzzles: Complete State Space $N = 3$



State space for $N = 4$ is $5 \cdot 10^7$ times bigger!

8-Queens Problem

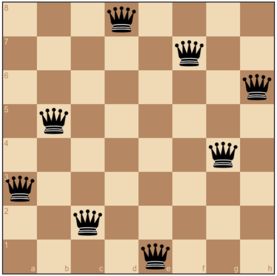
The goal is to place eight queens on a chessboard such that no queen attacks any other.

States: Any arrangement of 0 to 8 queens on the board.

Initial State: No queens on the board.

Actions: Add a queen to any square.

Goal: 8 queens are on the board, none attacked.



8-Queens Problem: Revision

Is the problem well formulated?

Any queen anywhere

$\approx 2.8 \cdot 10^{14}$ states $(n^2)^n$

Queens on different squares

$\approx 1.8 \cdot 10^{14}$ states $n^2!/(n^2 - n)!$

Queens in separate columns

$\approx 1.7 \cdot 10^{7}$ states n^n

Queens in separate cols, rows

40320 states $n!$

Searching the State Space

We have formulated some problems, how to solve them?

- A solution is a sequence of actions.
  - We are interested in a path to the goal state or the goal state itself.
- Search algorithm is searching for solution by applying each legal action to the current state. This is called **expansion**.
- How to choose which state to expand next is called **search strategy**.
- All possible action sequences form a **search tree** with the root corresponding to initial state.

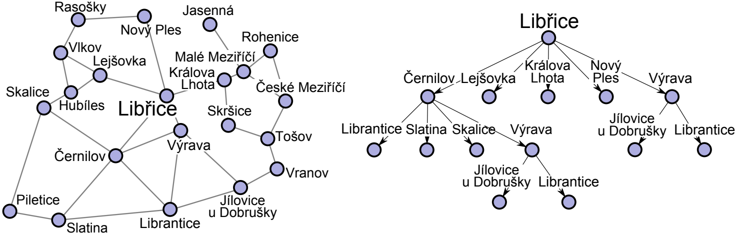
Search Tree

- Initial state at the root node.
- Edges are labeled with actions.
- Each node in the search tree corresponds to some path in the state graph.

Definition (The Search Tree)

Having state space $X = \langle S, A, \mathcal{I}, G \rangle$ with one initial state s_0, the search tree is a directed rooted tree with root s_0 such that each path in the tree exists in the graph (S, A).

Example of state graph and search tree for route finding problem:



Loops: Closed List

Algorithms that forget their history are doomed to repeat it!

We prevent exploring redundant paths by remembering already expanded nodes in **closed list**.

Each node can be in one of the following states:

- **FRESH** – not explored node,
- **OPEN** – explored, but not expanded node,
- **CLOSED** – expanded node.

Intuitively, already opened or closed nodes can be discarded from further exploration.

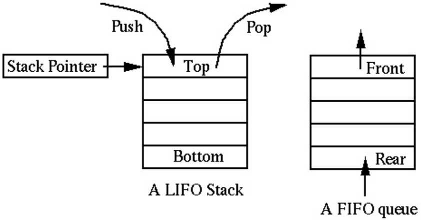
Implementation: Queues

Queue is a data structure to store elements (nodes). For queue is characterizing the order in which it stores the elements.

A FIFO queue , First-In First-Out.
Operations: enqueue, dequeue

A LIFO stack , Last-In First-Out.
Operations: push, pop, top

Priority queue pops the element of the queue according to some ordering function.



Search Strategy

Uninformed Search:

- no additional information about the states,
- goal-state or non-goal state.

Search strategy performance

- **Completeness:** If solutions exists does the strategy guarantee to find it?
- **Optimality:** Does the strategy find the optimal solution?
- **Time complexity:** How much time it takes?
- **Space complexity:** How much memory is needed?

For complexity estimation: **branching factor** b is the maximum number of successors of any node, d is the **depth** of the shallowest goal node, m is the maximum length of any path in the state space.

Informed (heuristic) search next lecture...

Random Search

- The simplest search algorithm.
- Algorithm chooses randomly the next node to be expanded.

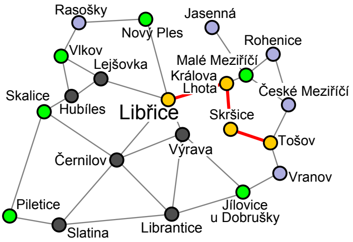
Complete: No

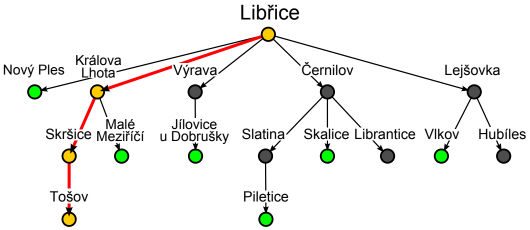
Optimal: No

Time: $O(b^d)$

Space: $O(b^d)$

Random search: Example





FRESH OPEN CLOSED INIT/GOAL EXPAND

Algorithm 1 Random search

```
open ← 𝓘;  closed ← {}
prev ← init_table()
while open ≠ {} do
    x ← random element of open
    if x ∈ G then
        return reconstruct_path(prev, x)
    end if
    for all y ∈ neighbors(x) do
        if y ∉ (open ∪ closed) then
            open ← open ∪ {y}
            prev[y] ← x
        end if
    end for
    open ← open \ {x};  closed ← closed ∪ {x}
end while
```

Breadth-First Search (BFS)

Simple strategy that expanding the search tree level by level. First, root node is expanded, then all the successors of root, then all their successors etc.

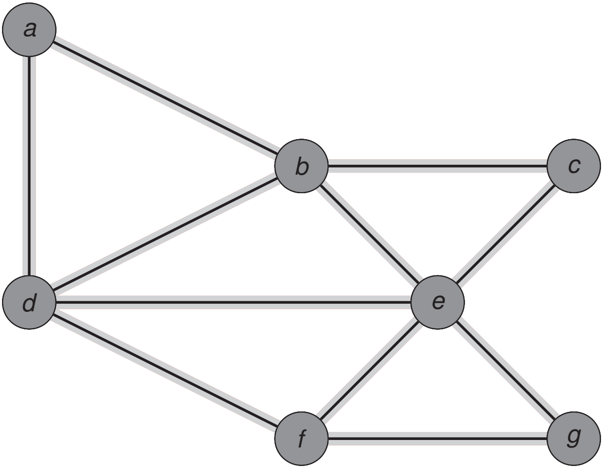
- Implemented using FIFO queue.

Complete: Yes

Optimal: Yes

Time: $O(b^d)$

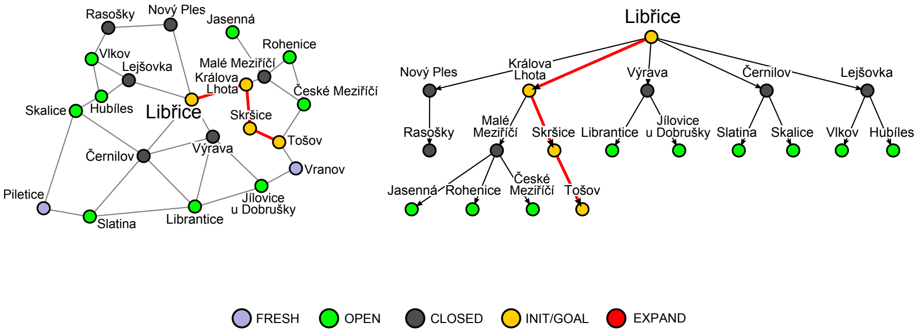
Space: $O(b^d)$


a
b
c
d
e
f
g

Algorithm 2 Breadth-First Search (BFS)

```
open ← init_queue(); closed ← {}
prev ← init_table()
for all s ∈ I do
    enqueue(open, s)
end for
while ¬empty(open) do
    x ← dequeue(open)
    if x ∈ G then
        return reconstruct_path(prev, x)
    end if
    for all y ∈ neighbors(x) do
        if y ∉ (open ∪ closed) then
            enqueue(open, y); prev[y] ← x
        end if
    end for
    closed ← closed ∪ {x}
```

Breadth-first search (BFS): Example



Depth-First Search

Idea of DFS is to expand in each step current node to travel as deep as possible.

- Implemented using LIFO stack.

Complete: No

Optimal: No

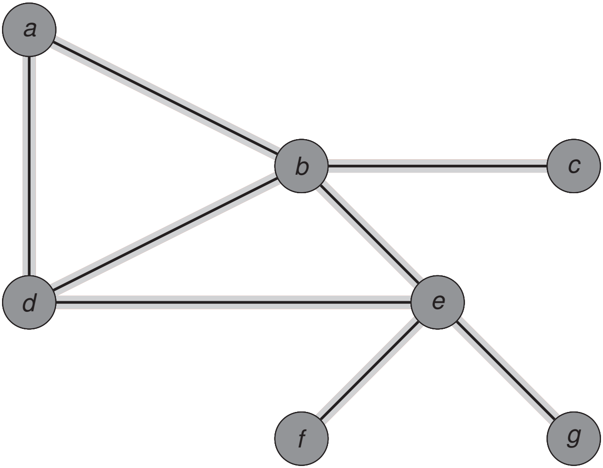
Time: $O(b^m)$

Space: $O(bm)$

Once a node has been expanded, it can be removed from memory as soon as all its descendants have been fully explored.

Extensions:

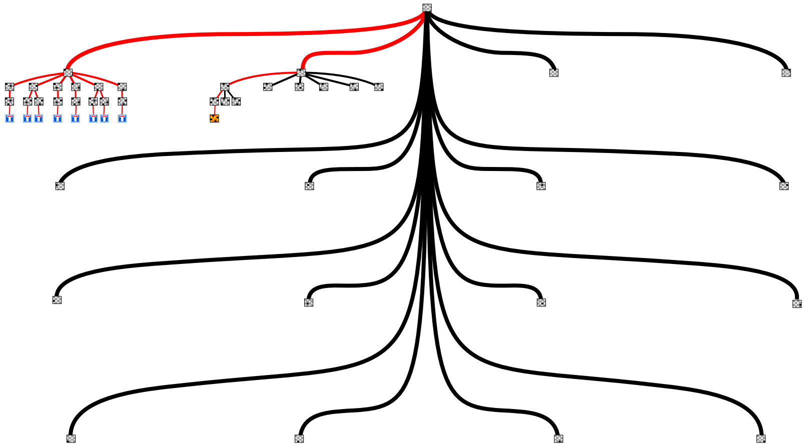
- Depth Limited Search
- Iterative Deepening Search (Complete, same time complexity as BFS, but space effective)


a
b
c
d
e
f
g

Algorithm 3 Depth-First Search (DFS)

```
open ← init_stack(); closed ← {}
prev ← init_table()
for all s ∈ I do
    push(open, s)
end for
while ¬empty(open) do
    x ← pop(open)
    if x ∈ G then
        return reconstruct_path(prev, x)
    end if
    for all y ∈ neighbors(x) do
        if y ∉ (open ∪ closed) then
            push(open, y); prev[y] ← x
        end if
    end for
    closed ← closed ∪ {x}
```

Depth-first search (DFS): 4 Queens problem



Path Reconstruction Algorithm

- Same for all the algorithms.
- Each node remembers its predecessor.

Algorithm 4 reconstruct_path(*prev*, *goal*)

```
x ← goal
path ← init_list()
while x ≠ NULL do
    append(path, x)
    x ← prev[x]
end while
return path
```